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III.

THE DYNAMICAL EFFECTS OF AGGREGATES OF ELECTRONS.

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I. ELECTRONS AND MATTER.

The enormous difference in the behavior of different materials towards electric force is a matter with which everyone is familiar; and it is one of the triumphs of the electron theory that it has given us a very satisfactory picture of the difference between insulators and conductors of electricity. We are to regard all matter as made up primarily out of electrons. They are the stones with which the material structure is built up, the electrodynamic forces are the cement which holds the stones together. There are, however, two different ways in which the electrons may exist in a given portion of matter. They may be located in position of stable equilibrium, in which case a very small force will displace them to a small extent but a perfectly enormous force would be required to dislodge them thoroughly and give rise to instability; or they may be so loosely held that they are able to move about in the interstices of the material, very much after the fashion in which we believe the molecules move about in a gas. In the former case, when the electrons are practically fixed, we say the substance is an insulator; in the latter case, where they are wandering about, the substance is a conductor.

A moment's reflection will show that this difference is sufficient to explain the difference between insulators and conductors. Consider what happens when a slab of the first kind is placed in an electric field. There will be a displacement of the electrons, it is true, but the displacement will be small and they will all return to their original equilibrium positions as soon as the external field is

removed. There will be no transportation of electrons and that is what, on the electron theory, constitutes an electric current. The electric field across the slab is, nevertheless, different from what it would be if the material were not present. The difference between different insulating materials in this respect depends solely on the comparative ease of displacement of the electrons they contain. The specific inductive capacity of dielectrics, which, you will remember, was discovered by Cavendish and Faraday, is, in fact, a measure of the product of the number of electrons in unit volume of the material by the average displacement which they undergo in unit field.

The behavior of the second kind of material is quite different. Even in the absence of the electric field, the so-called free electrons are moving about in it in an irregular manner in all directions. The effect of an external field is to superpose on the irregular motion a definite drift, on the average, in the direction of the current. This drift of the electrons involves *transportation* of electricity and therefore implies the existence of an electric current.

All the laws which regulate the transference of electricity across conductors, such as, for example, Ohm's Law, which states that the current is proportional to the applied electromotive force, and Joule's Law, which states that the rate of production of heat by a current is equal to the product of the resistance of the circuit by the square of the current, follow at once from this simple hypothesis. It is not necessary to suppose that all the electrons in the material are present in the free condition; some of them may be, and in all probability the majority are, in a state of equilibrium similar to that which occurs in insulators. All that is necessary is that some of the electrons should be able to move without restraint. When the other conditions are the same the magnitude of the current which a given material will transport is proportional to the number of carriers available; that is, to the number of free electrons per unit volume.

It is in the explanation of the relation between the conductivity of substances for electricity and for heat that the electron theory has scored one of its most notable triumphs. Everybody knows that the best conductors for electricity are also the best conductors for heat. It is not so generally known how very close the relationship

between the two phenomena is. In the accompanying table two columns of figures are shown.

Material.	Ratio: Thermal Conductivity, Electrical Conductivity.	Temperature Coefficient of this Ratio, Per Cent.
Copper, commercial	6.76×10^{10} at 18° C.	—
Copper (1) pure.....	6.65×10^{10} at 18° C.	.39
Copper (2) pure.....	6.71×10^{10} at 18° C.	.39
Silver, pure	6.86×10^{10} at 18° C.	.37
Gold (1)	7.27×10^{10} at 18° C.	.36
Gold (2) pure.....	7.09×10^{10} at 18° C.	.37
Nickel	6.99×10^{10} at 18° C.	.39
Zinc (1)	7.05×10^{10} at 18° C.	.38
Zinc (2) pure.....	6.72×10^{10} at 18° C.	.38
Cadmium, pure	7.06×10^{10} at 18° C.	.37
Lead, pure.....	7.15×10^{10} at 18° C.	.40
Tin, pure	7.35×10^{10} at 18° C.	.34
Aluminium	6.36×10^{10} at 18° C.	.43
Platinum (1)	7.76×10^{10} at 18° C.	—
Platinum (2) pure	7.53×10^{10} at 18° C.	.46
Palladium	7.54×10^{10} at 18° C.	.46
Iron (1)	8.02×10^{10} at 18° C.	.43
Iron (2)	8.38×10^{10} at 18° C.	.44
Steel	9.03×10^{10} at 18° C.	.35
Bismuth	9.64×10^{10} at 18° C.	.15
Constantan (60Cu, 40Ni)	11.06×10^{10} at 18° C.	.23
Manganin (84Cu, 4Ni, 12Mn)...	9.14×10^{10} at 18° C.	.27

They represent the results of measurements by Jaeger and Dieselhorst of the electric and thermal conductivities of a large number of metals and alloys. The first column of figures gives the ratio of the thermal to the electrical conductivity for each of these substances and the second gives the percentage change of this ratio when the temperature is increased one degree. It will at once be noticed that the numbers in each column are almost equal, particularly if we keep to the pure metals. Thus for every pure metal the electrical conductivity bears to the thermal conductivity a proportion which is almost independent of the metal: and the ratio of the thermal conductivity to the electrical conductivity increases by almost the same amount for one degree rise of temperature for each metal. The coefficient of increase of this ratio with increase of temperature is also very nearly equal to the coefficient of increase of the volume of all gases with temperature, when the pressure is maintained constant.

These interesting relations were shown to be a consequence of the electron theory of conductors by Drude. He proved that they follow inevitably from the assumptions (1) that a metal contains electrons which move about freely like the molecules of a gas, (2) that they possess a certain average mean length of free path λ during the traversing of which they are only acted on by the external applied electric force, (3) that this path is terminated by a collision and that the new motion which then ensues is, on the average, independent of the previous motion; and lastly (4) that their average kinetic energy is the same as the average kinetic energy of translation of a molecule of any gas at the same temperature as the metal.

A simplified form of Drude's deduction may be given here. If X is the electric intensity inside the metal, e the electric charge possessed by an electron and m its mass, then the force acting on the electron during its free path is Xe and its acceleration Xe/m . If the velocity of the particle at the beginning of the path is u its velocity at the end will be $u + \frac{Xe}{m}t$ where t is the average time between two collisions. The average velocity in the direction of the electric field is therefore $\frac{1}{2}X\frac{e}{m}t$ since the average value of u taken over a large number of electrons is zero. Now the free path λ is equal to vt where v is the mean speed. Thus the average drift velocity of the electrons in the direction of the electric field may be written in the form $\frac{1}{2}X\frac{e}{m}\frac{\lambda}{v}$. If n is the number of electrons in unit volume, the number of them which, in unit time, drift across a unit area drawn perpendicular to the direction of the electric force X will be $\frac{1}{2}nX\frac{e}{m}\frac{\lambda}{v}$. Each of these carries a charge e so that the quantity of electricity transported across unit area, or in other words, the electric current density will be

$$I = \frac{n}{2} \frac{e^2}{m} \frac{\lambda}{v} X.$$

Now it is a necessary consequence of the principles which underlie the kinetic theory of matter that $\frac{1}{2}mv^2$ should be equal to $\alpha\theta$ where θ is the absolute temperature and α is a universal constant which may

be calculated from the properties of gases. This assertion is the mathematical statement of the relation (4) enumerated above. Making this substitution we find that the specific electrical conductivity of the material is $\sigma = \frac{i}{X} = \frac{ne^2\lambda v}{4\alpha\theta}$. In this formula e^2 and α have the same value for all substances, n and λ are constants characteristic of each substance, v is independent of the nature of the material but is proportional to the square root of the absolute temperature.

It is a well-known result of experiment that the specific conductivity of all substances is inversely proportional to the absolute temperature. We therefore conclude that the product $n\lambda$ for all metals must be inversely proportional to the square root of the absolute temperature.

It is a well-known result of the kinetic theory of gases that the thermal conductivity of a gas is equal to $\frac{1}{3}n\lambda v\alpha$. Hence $\frac{k}{\sigma} = \frac{4\alpha^2}{3e^2}\theta$. Thus this ratio should have the same value for all metals at the same temperature and the temperature variation should be the same as that of the volume of a gas at constant pressure. These are the relations which are exhibited by the experimental results of Jaeger and Diesselherst.

The electron theory of metallic conduction has enabled us to understand a number of curious effects which occur when a conductor is placed in a magnetic field. One of these, the Hall effect, consists in a deflection of the line of flow of a current which is caused by the magnetic field. Another effect, which is especially marked in the case of Bismuth, is an alteration of the specific resistance of the material caused by a magnetic field. These effects are intimately connected together and have a simple explanation on the electron theory. It is well known that any electrified particle moving in a magnetic field is acted on by a force which is perpendicular to the plane containing the magnetic force and the direction of motion. The superposition of this force upon the other forces acting on the electrons in a metal carrying a current will cause all the electrons to curve round in the same general direction, giving rise to the Hall effect. It will also increase the average curvature of the paths of the

electrons. In this way the time which is required for electricity to be transferred will be greater so that the specific electrical conductivity will be diminished. This is the explanation of the second effect. Both these effects are complicated by the action of the electrons on the atoms so that the foregoing description is only to be regarded as a rough outline of what really occurs.

So far we have only considered the way in which the electron theory of conduction explains a number of phenomena which were familiar before it was enunciated. The power to do this is a necessary attribute of every scientific theory. A scientific theory, however, is often much more useful than this in that it leads to the prediction of phenomena which would hardly have been foreseen without its aid. The present theory has been able to prove its usefulness in this way, as the principles underlying it have enabled us to develop a new chapter in physical science, a chapter to which I have ventured to give the name of Thermionics. Thermionics relates to the emission of electrified particles by hot bodies and the phenomena to which they give rise.

It is found that all bodies when heated to a sufficiently high temperature give rise to an emission of both negatively and positively charged particles. In many ways the negative emission is the more interesting as the particles emitted are negative electrons having properties identical with those of the carriers of the cathode rays. The connection between this emission of negative electrons and the transportation of electricity in a metallic conductor is very intimate. We have seen that, in order to explain the phenomena exhibited by metallic conduction, it is necessary to suppose that such conductors contain large numbers of "free" electrons. If these electrons are moving about freely inside the conductor, as we have supposed, the question at once arises as to why they do not escape into the surrounding atmosphere. It is clear that they do not do so, otherwise there would be a leakage of electricity from the surface of all conductors at ordinary temperatures. The answer must be that there are forces at the surface of the metal which are sufficiently great to prevent them from escaping. Now consider what we should expect to happen as the temperature of such a body is raised. We have supposed that the average kinetic energy of the contained electrons

is higher the higher the temperature. Clearly, at a sufficiently high temperature some of the particles will have enough energy from their heat motion to be able to break through the surface. Moreover, the number which are able to escape will be greater the higher the temperature.

A theory following these lines has succeeded in predicting the way in which the emission of the electrons depends upon the temperature as well as a number of other interesting relations between the thermal and electrical behavior of substances. It will be remarked that the view which has been outlined is very similar to the view of the phenomenon of evaporation which is afforded by the kinetic theory of matter. According to that theory the particles of the liquid escape into the vapor when their kinetic energy (to be accurate we ought to say that part of it which depends on the component of velocity normal to the surface) exceeds the work they have to do in order to pass through the surface. Thermionic emission may be looked upon then simply as the evaporation of electrons which may be regarded as dissolved in the metal. Just as water is cooled when it evaporates and heated when steam condenses into it; so we should expect a conductor to be cooled when it emits electrons and heated when it absorbs them. Both these effects have recently been discovered, the former by Wehnelt and Jentzsch and the latter by Richardson and Cook.

There is one point in this connection which is worthy of further consideration. We have seen that it is necessary to suppose that the electrons in a metal behave like the molecules of a gas. The same will be at least as true of them after they have been emitted. Thus when a metal at a high temperature lies in an air-tight enclosure there will be two atmospheres of electrons, one at a high pressure inside the metal and the other at a low pressure in the enclosure outside of the metal. If the principles of the kinetic theory of matter are well grounded it can be shown that in both of these atmospheres the electrons are moving about with all possible speeds but that the proportion of them which have a given speed is the same for each atmosphere. Moreover, the proportion is the same known function of the temperature in each case and in each case also

the average kinetic energy should be the same as the average kinetic energy of a molecule of any gas at the temperature of the enclosure.

In fact the laws of the kinetic theory of gases can be applied without change to the atmospheres of electrons; and the above assertions are simply statements of a theorem in the kinetic theory of gases called, after its discoverers, the Maxwell-Boltzmann Law. According to this law if a large number of molecules are selected at random out of any gas the proportion of them which have speeds lying between certain assigned values let us say u and u' is a certain definite function of u and u' . The value of this function, which in addition to u and u' depends only upon the temperature of the gas and the mass of its molecules, was first deduced by Maxwell. Maxwell's deduction of the value of this function, though sufficiently convincing to those who are familiar with the methods of mathematical physics, was, nevertheless, a highly abstract piece of reasoning; and it has been impossible up to the present to make anything in the nature of a direct test of it by experiment on gases. With the atmospheres of electrons we are, however, able to do a great deal more than we could with a gas made up of uncharged molecules. By placing them in a suitable electric field we can bring forces to bear on each individual electron which are enormous compared with the forces exerted on a molecule by the earth's gravitational field. For example if the electrons are being emitted from a heated flat plate we can place another flat surface a little in front and charge it up, so that the electric field tends to drive the ions back into the surface at which they originated. Under these circumstances only those electrons will be able to cross from one plate to the other if their kinetic energy is greater than a certain value depending on the electric field between the plates; thus the current that gets across will be a measure of the number of electrons emitted whose kinetic energy exceeds a known value. By experiments of this kind, and others based on similar principles, we have succeeded in determining the law of distribution of speed among the individual electrons which are emitted. It is found to agree in every particular with that predicted by Maxwell for the case of a gas whose temperature is the same as that of the metal emitting the electrons and whose molecular weight is equal to the mass of an electron. In particular the average

kinetic energy of the electrons is the same as that of the molecules of a gas at the temperature of the metal which emits them; and we can calculate the value of the well-known constant R in the gas equation $pv = R\theta$, where p is the pressure, v the volume and θ the absolute temperature of the gas, from purely electrical experiments of the kind indicated. It follows from the results of these experiments together with a simple application of the principles of the dynamical theory of gases that the free electrons inside a metal must have the distribution of velocity which is required by Maxwell's law and in particular must have the same average translational kinetic energy as the molecules of a gas at the temperature of the metal which contains them.

2. MATERIAL MEDIA AND ELECTROMAGNETIC RADIATIONS.

The action of light on insulating media is a rather complicated, but extremely important, phenomenon on which the electron theory has thrown a great deal of light. Maxwell showed, many years ago, that light is an electromagnetic phenomenon. A beam of light is in fact a wave of oscillating electric and magnetic force, the electric and magnetic forces being at right angles to one another and to the direction of propagation. When such a wave falls on an insulating medium the oscillating electric force will set into vibration the comparatively stable electrons which, as we have seen, are embedded in the medium. The electrons will execute what are appropriately called forced oscillations, about their original equilibrium positions, and these oscillations will have the same periodic time as the light. Thus when it traverses a material insulating medium the light has not only to keep itself going; it has to keep the electrons which make up the medium going as well. Roughly speaking one may say that the electrons in such a medium behave like a load on the luminiferous ether. We should therefore expect them to diminish the speed of propagation of light through it and this is found to be the case. The exact expression for the velocity cannot be obtained without going more deeply than we have time to into the electromagnetic theory of light. It was first given by Maxwell, who showed that the refractive index, to which the velocity of propagation is inversely proportional, was equal to the square root of the

effective specific inductive capacity of the medium. Now the specific inductive capacity of an insulating medium is equal to unity plus the product of the number of electrons per unit volume by their average displacement in unit electric field. When the material is subjected to constant electric forces the displacements of the electrons are always proportional to the forces and the specific inductive capacity is therefore a constant quantity. When the force is an oscillating one the matter is complicated by the fact that the electrons try, as it were, to strike a balance between their own natural period of oscillation and that of the force acting on them. They end by oscillating with the same frequency as the force which excites them but the distance they travel from their equilibrium position depends a good deal on their natural periods as well. Thus the specific inductive capacity for oscillating forces will not be a constant quantity but will depend to some extent on the frequency of oscillation of the force. By the effective specific inductive capacity we mean the specific inductive capacity for electric forces which oscillate with the frequency of the light under consideration.

It is evident from what has been said that the refractive index of an insulating substance depends upon the frequency or, in other words, upon the color of the light. We see at once why a beam of white light is split up by a prism into the constituent spectral colors. For each ray is deviated by the prism according to the value of its refractive index.

Perhaps the most interesting question in this part of our subject is that of the behavior of a substance towards light whose frequency is close to that of the natural periods of the substance. In that case the electrons are set into violent motion owing to the occurrence of what are sometimes called sympathetic vibrations. The nature of this phenomenon may best be illustrated by considering a simple mechanical analogy. Imagine a spiral wire with a weight at one end to be hung from a shaking support. If the weight is pulled down and let go it will oscillate backwards and forwards with a definite natural frequency which depends on the stiffness of the spring and the heaviness of the weight. If the shakiness of the support arises from tremors in the building, to the walls of which we will suppose it bolted, as a rule the frequency of its vibrations will

be very great compared with the natural frequency of the spring. In that case the shakiness of the support will have very little effect on the spring. If however the frequency of the tremors happens to be equal or nearly equal to the natural frequency of the spring the latter is set into very violent agitation, for the reason that the natural swings of the spring are continually being helped by the oscillations of the support.

A precisely analogous effect takes place when the period of the light is close to the natural period of the electrons. In fact it can be shown that, if there is nothing analogous to a frictional force to damp down the vibrations of the electrons, they will execute oscillations of infinite amplitude when there is exact coincidence between the periods. Since the displacement of the electrons in unit electric field is *the* important factor in determining the refractive index we should expect its value to change very considerably in this region. As a matter of fact, in the extreme case where there is no damping, the value of μ^2 falls rapidly from a small positive quantity on the short wave-length side of the position of coincidence to the value $-\infty$ at exact coincidence ($\lambda = \lambda_0$). As the period of exact coincidence is passed μ^2 changes suddenly to $+\infty$ and on the long wave-length side falls rapidly to a rather larger positive value than the one that it had at a great distance from the natural period on the short wave-length side.

Several very important deductions can be drawn from the results which have just been described. In the first place we notice that provided we always keep to the same side of the natural period, no matter which side we choose, then the refractive index μ always diminishes as the wave-length λ increases. Hence, since the deviation of light by a prism is greater when the refractive index is greater it will be smaller the greater the wave-length. The blue light will therefore be deviated more than the red light in the spectrum. This is the well known kind of dispersion which is exhibited by prisms of glass and similar colorless transparent substances.

When part of the spectrum lies on one side of the natural period and part on the other there is a sudden increase in the value of the refractive index when the natural period is crossed. The spectrum will then consist of two groups of colors, that on the long

wave-length side being more deviated than that on the short wave-length side, although in each group the colors are in the normal order. This is the so-called anomalous dispersion which was discovered by Kundt and which is exhibited by all transparent colored bodies, like the aniline dyes, which possess a metallic shimmer. Immediately on the short wave-length side of λ_0 , we have seen that μ^2 has a negative value in the case we are contemplating. μ in this region has therefore what mathematicians call an imaginary value. It can be shown that this imaginary value means that the waves are incapable of entering the medium. When a train of waves of this wave-length falls on the medium they are not absorbed, properly speaking, but are completely reflected. The substance would appear to be opaque to light of this wave-length not because it absorbs the light which falls on it but because it reflects it completely. If mixed light which contained some of this particular wave-length were made to undergo a sufficient number of successive reflections from plates of the substance, only light of this particular region of frequency would ultimately be left over, since a certain percentage of the other wave-lengths always gets through. This principle has been utilised by Rubens to isolate radiations of definite wave-length in the infra-red part of the spectrum. These radiations are called, very appropriately, residual rays.

The foregoing discussion does not touch the very interesting questions of the absorption of light by insulating media. There will be no absorption, properly speaking, unless there are forces acting on the moving electrons which tend to dissipate the energy of the light. Such forces must in general exist and it is usually assumed that there is a retarding force proportional to the velocity of the moving electrons, chiefly because this is the simplest assumption which can be made which is not in contradiction with fact. The existence of forces of this kind modifies the foregoing conclusions to a considerable extent in detail but it does not affect their general character.

Planck has pointed out that it follows from the principles of the electromagnetic theory of light that the radiation from the moving electrons gives rise to a retarding force which may be taken to be proportional to their velocity. Such a force must unquestionably

exist but its magnitude is quite small. It is of interest to see if it is sufficiently large to account for the known cases in which the dissipation of energy is smallest. These are unquestionably the cases in which residual rays are obtained. I have developed a formula which expresses the percentage of incident energy which goes into the residual rays, which includes the case where dissipation is taken account of. This formula leads to two separate methods of estimating the order of magnitude of the dissipation. Both these methods show that the dissipation must be of the order of 10^{12} in certain units where Planck's theory leads to a dissipation of the order 10^4 in the same units. Thus the source of dissipation pointed out by Planck is about 10^8 times too small to account for the smallest case of dissipation known to us.

I am inclined to think that the most general type of absorption of light by bodies of this class is of the following character. We have seen that the electrons execute forced vibrations under the influence of the incident light. When the period of the light approximates to the free period of the electron the electrons absorb a great deal of energy from the light. In general this absorption of energy will go on until the vibrations carry the electron out of its region of stability. A rearrangement of the system will then take place and during this rearrangement a great deal of the kinetic energy which the electron has accumulated will be transferred to other parts of the substance and will make itself felt as heat. As far as that particular electron is concerned the sympathetic vibrations will have to be established all over again. It is not necessary to suppose that during this process the electron is actually carried out of the atom when it breaks loose from the region of stability. The whole occurrence may take place in the one atom. On the other hand we know a great many cases of bodies which emit electrons under the influence of light and in these cases the electrons must get carried out of the atom. It seems to the writer to be an advantage of this view that it connects the absorption of light with the so-called photo-electric effect. As a first approximation this view of the absorption of light leads to the same relation between absorption and frequency as does the assumption of a retarding force proportional to the velocity.

There is another point in this connection that is not without interest. On any theory of absorption the natural periods of a substance are characterised by conferring on it either intense absorption or intense opacity. It is therefore evident that they can be detected very readily by experiment. From an analysis of the natural periods of a large number of substances which has been carried out by Drude it appears that there are two types of vibrations which occur. In the one the electron forms the vibrating system and in the other one of the constituent atoms or a group of atoms vibrate as a whole. Owing chiefly to the enormous difference between the mass of an electron and that of an atom there is an enormous difference between the frequency of the two types. The electronic type always gives rise to frequencies in the ultra-violet part of the spectrum and the atomic type to natural frequencies in the infra-red. It is therefore not an accidental circumstance that almost all chemically pure substances which are not conductors of electricity are transparent in the visible spectrum.

The action of Roentgen rays on matter is a subject of great interest. According to the ether pulse theory of these rays elaborated by Sir J. J. Thomson, the relation between the Roentgen rays and sodium light is similar to that between a series of sharp cracks and a musical tone. And on the modern view of the nature of white light the difference between white light and the Roentgen rays is one of degree rather than kind. The cracks corresponding to Roentgen rays are much sharper than those which correspond to white light. According to the principles of harmonic analysis which we owe to Fourier it should be possible to resolve either of these kinds of radiation into simple harmonic elements. I have estimated that the average frequency of these elements for the Roentgen rays would be 10,000 times greater than that for those which form white light. This estimate is based on the view that the kinetic energy of the electrons emitted by bodies under the action of ultra-violet light and Roentgen rays is a function of the frequency of the equivalent vibrations. The experimental results indicate that the functionality is a linear one and there is considerable theoretical support for this view. Some investigators have maintained that the square root of the energy is proportional to the frequency; but even if this extreme

view is taken the estimated frequency is not changed enough to affect the general argument.

If the Roentgen rays are so much like white light you will at once ask why they are not deviated by a prism. The answer is very simple. It follows from the principles of the electron theory that the refractive index μ of a substance for electromagnetic vibrations

of frequency p is given by $\mu^2 = 1 + \Sigma \frac{\nu_s e^2}{m(p_s^2 - p^2)}$ where e is the charge on an electron, m its mass, p_s its natural frequency and ν_s the number of electrons of type s in unit volume of the material. In general this formula will not be exact on account of the interactions of the electrons on one another but it will give results of the right order of magnitude. Now e and m have the same value for all electrons and in the part of the spectrum near the visible $\nu_s e^2 / mp_s^2$ is of the order unity for such frequencies p_s as fall within that region. Now ν_s will always be of the order of the number of molecules per cubic centimeter whichever of the s classes of electrons we consider, so that we may draw the following conclusions: (1) On account of the very great absolute value of p , μ^2 will be equal to unity except for a very narrow range in the immediate neighborhood of $p = p_s$. (2) Only such substances will be capable of refracting the Roentgen rays as have natural frequencies p_s which lie within the range of values of p embraced by the Roentgen rays experimented on. In any event it is clear that with a mixed group of rays such as is emitted by an ordinary X-ray bulb, practically the whole of them will pass through a prism without deviation.

Barkla's experiments on secondary rays show that the Roentgen rays exhibit phenomena very much akin to fluorescence in optics. One interpretation of Barkla's results would be that there are in material atoms natural frequencies comparable with the frequencies in the Roentgen rays. In that case, although almost the whole of a beam of Roentgen rays would be undeviated by a prism there should be a small amount which would be deviated. At present I am making experiments to detect this effect.

Of course if one adopts the corpuscular view of the Roentgen rays recently developed by Bragg, effects of the kind described are

not to be expected. At present the balance of evidence seems to be decidedly against the corpuscular view.

I am inclined to think that the primary Roentgen rays originate largely as the result of secondary actions due to the stirring up of the electrons in the atoms of the anti-cathode by the rapidly moving cathode rays which impinge on them. On this view the constituent frequencies of the rays would be, to a considerable extent, a matter of the atoms in which they originate; and it may be that the gap in electro-magnetic radiations between ultra-violet light and the Roentgen rays, which exists at present, may never be filled up; as there may be no atoms which have natural periods in the neighborhood of these frequencies.

Recent years have seen the accumulation of a very large quantity of material relating to optical effects which are produced by a magnetic field. It is impossible, within the limits of this discussion, to attempt to show the enormous usefulness of the electron theory in the development of the science of magneto-optics; but there is one phenomenon which we cannot afford to pass by entirely, if only on account of its historical importance. I refer to the Zeeman effect. This effect was called after its discoverer, who showed that the spectral lines which are emitted by all gaseous substances under suitable conditions of excitation, were slightly displaced by a very strong magnetic field. The true explanation of this phenomenon was at once given by H. A. Lorentz. He pointed out that if the monochromatic light was emitted by vibrating electrons the frequency of the vibrations would be altered if the atom which contained the electrons found itself in a magnetic field. This change in the frequency, of course, corresponds with a change in the wavelengths of the emitted light. He also predicted that the emitted light would be polarized in a certain way and this was confirmed by experiment. Lorentz showed, in addition, that the value of the electric charge of an electron, divided by its mass, could be calculated from the displacement of the spectral lines in the magnetic field. The results of these calculations showed that the value of this ratio was the same as that found by Sir J. J. Thomson and Wiechert for the cathode rays in a discharged tube. Thus the Zeeman effect and the cathode rays were the first two phenomena which afforded ex-

plicit evidence of the existence of these minute charged particles, whose mass is nearly 2,000 times smaller than that of the lightest known chemical atom.

The emission of ordinary heat radiation such as is given out by all substances, and in increasing amount the higher the temperature, is very intimately connected with the theory of electrons. As is well known this radiation is electromagnetic in character and includes visible light as a particular case. We also know that when an electron is accelerated it emits electromagnetic radiation. It is natural therefore to attribute the origin of this radiation to the motions of the electrons of which material bodies are made up. By making use of the principles of thermodynamics we can prove that the nature of the radiation of this character which is to be found in any enclosure maintained at a given temperature is independent of the nature of the walls of the enclosure. The amount and character of this radiation is thus independent of the material from which it originates. If therefore we can calculate the amount of this radiation of each wave-length for any particular substance, for a series of temperatures, we shall know what it is for all substances at the same temperatures. Unfortunately when such a calculation is carried out in the most logical and natural way it leads to results which are not in agreement with those given by experimental measurements. Another mode of calculation given by Planck leads to a formula which agrees with the experimental values. It has been shown however that Planck's calculation involves the implicit assumption that energy is an atomic or discontinuous quantity. This idea is distasteful to many physicists and it is so revolutionary that it is not desirable to adopt it without very convincing evidence. One of the authorities on this very intricate subject, Jeans, maintains that the reason for the discrepancy between the less revolutionary theory and experiment is due to the fact, as he asserts, that the experiments do not measure the true equilibrium radiation. However this may be, the difficulties which lie in the explanation of the connection between radiation and temperature do not belong to the electron theory proper but are outside of it. No matter how it may be decided the outcome of this question is not likely to shake the foundations of the electron theory of matter.

3. THE NUMBER OF ELECTRONS IN AN ATOM.

The behavior of very rapidly moving electrons in their passage through matter is a very interesting subject of investigation. Thanks to the discovery of the radio-active substances we are able to experiment, if we wish, with electrons whose speed is almost equal to that of light (3×10^{10} centimeters per second). These rapidly moving electrons are able to shoot right through the atoms of bodies, but when in their flight they pass very close to one of the constituent electrons their paths are deviated. When a group of them passes through a considerable thickness of matter these deviations tend to accumulate; so that a group of electrons, all of which were moving parallel to one another to start with, becomes divergent. Sir J. J. Thomson showed how the average deviation arising from a single impact could be calculated and also how the average divergence of the beam, which was caused by its passage through a given amount of matter, would depend on the number of electrons present in the matter traversed.

Experiments made by Barkla showed that when matter, which was made up of elements of low atomic weight, was traversed by Röntgen rays it was caused to emit so-called secondary Röntgen rays, which were precisely similar in character to the primary Röntgen rays which excited them. A careful study of these secondary rays showed that they were primary rays which had been scattered. The phenomenon is in fact very analogous to that which gives rise to the blue color of the sky, which was shown by Lord Rayleigh to arise from light scattered by innumerable small particles present in the atmosphere. The amount of such scattering depends on the number of particles which are engaged in doing it. In the case of the Röntgen rays these particles are the electrons present in the matter, each one of which is set into violent motion by the Roentgen ray pulse. The exact way in which the amount of the scattering should depend on the number of electrons engaged in the operation was figured out by Thomson who showed from Barkla's experimental results that the number of electrons reckoned per atom of the material was comparable with the atomic weight.

We have seen already that information of a like nature may be

obtained from the spreading out of a beam of rapidly moving electrons when they traverse a layer of matter. Working over such material as was then available Thomson found his conclusion, that the number per atom was comparable with the atomic weight, was strengthened. Quite recently J. A. Crowther has made a very careful experimental investigation covering both these lines of inquiry. He finds that Thomson's calculations are borne out very satisfactorily by his experiments and that the number of electrons which go to make up each atom is three times the atomic weight of the element.

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